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Fachbereich Mathematik

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Nichtlineare Optimierung

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Übungsaufgaben 5

Problem 1. For $1 \leq i \leq n$ fix tuple (α_i, β_i) s.t. $\alpha_i < \beta_i$. Consider the set

$$\mathscr{X} = \{\mathbf{x} \in \mathbb{R}^n;\, \alpha_i \leq x_i \leq \beta_i \quad (1 \leq i \leq n)\}\,,$$

and consider the optimization problem:

$$\min_{\mathbf{x} \in \mathscr{X}} f(\mathbf{x})\,.$$

a) Show that if $\mathbf{x}^* \in \mathscr{X}$ is a local minimum, then

$$\begin{cases} \frac{\partial f}{\partial x_i}(\mathbf{x}^*) \geq 0 & \text{if } x_i^* = \alpha_i\,, \\ \frac{\partial f}{\partial x_i}(\mathbf{x}^*) \leq 0 & \text{if } x_i^* = \beta_i\,, \\ \frac{\partial f}{\partial x_i}(\mathbf{x}^*) = 0 & \text{if } \alpha_i < x_i^* < \beta_i\,. \end{cases} \tag{1}$$

b) In addition, suppose that f is convex. Then show that (1) is also sufficient for the optimality of $\mathbf{x}^*$.

Hint: For **a)** use the necessary optimality condition

$$\big\langle \nabla f(\mathbf{x}^*), \mathbf{x} - \mathbf{x}^* \big\rangle \geq 0 \qquad \forall\, \mathbf{x} \in \mathscr{X}\,.$$

Problem 2.

a) Let $\mathcal{A}, \mathcal{B} \subset \mathbb{R}^n$ be two convex sets. Show that $\mathcal{A} \cap \mathcal{B}$ is convex.

b) The convex hull $\mathscr{H}(\mathcal{A})$ of a set $\mathcal{A} \subset \mathbb{R}^n$ is the collection of all convex combinations of points from $\mathcal{A}$. Show that $\mathscr{H}(\mathcal{A})$ is the smallest convex set containing $\mathcal{A}$.

Abgabe: 22.05.2025.