

## 9. Sheet for Numerics of Stationary Differential Equations

### Programming Exercise 4:

Solve with the finite elements method the problem

$$\begin{aligned} -d\Delta u + cu &= f & \text{in } \Omega, \\ u &= 0 & \text{auf } \partial\Omega, \end{aligned}$$

where the domain  $\Omega$  and the parameters  $d > 0, c \geq 0$  are given as the following:

$\Omega$ : Direct as a triangulation through the matrices (elements, nodes);

$\partial\Omega$ : Through the list of boundary nodes (boundary)

$f$ : As a function (`func_f.m` or `func_f.jl`).

Implement a function `matrix_assembly(Elements,Nodes)` for the computation of the stiffness- and mass-matrix. Use the exercises 23, 24 and 25. The vector  $b$  can be approximated by

$$b|_j = \int_{\Omega} f \phi_j \approx \int_{\Omega} I_h f \phi_j = (Mf)_j$$

An example code for handling the txt-files can be found under: [https://na.uni-tuebingen.de/ex/num3\\_ws25/PA4\\_FEM.zip](https://na.uni-tuebingen.de/ex/num3_ws25/PA4_FEM.zip).

- (a) Solve with the help of your `matrix_assembly` function the linear equation system, which corresponds to the problem above and  $\Omega$  being the unit circle. A reference solution (`func_solution`) and the inhomogeneous function (`func_f`) are given as a m-file.

Compute the error of the numerical solution for different grids. The triangulations for the unit circle are encoded in the txt-files

$$\begin{array}{ll} \text{Elements\_j.txt} & j = 1, 2, 3, 4. \\ \text{Nodes\_j.txt} & \end{array}$$

- (b) Compute the error in the  $L^2$  norm and the  $H^1$  semi-norm through:

$$\begin{aligned} \|e_h\|_{L^2(\Omega)}^2 &= \|\mathbf{e}\|_M^2 = \mathbf{e}^T M \mathbf{e}, \\ \|\nabla e_h\|_{L^2(\Omega)}^2 &= \|\mathbf{e}\|_A^2 = \mathbf{e}^T A \mathbf{e} \end{aligned}$$

and plot the errors (loglog-plot!).

**Solutions are discussed on Tuesday January 13, 2026**

**Tutor: Georgios Vretinaris - if you have question just come to my office (C3P16) or write me an email.**



**We wish you all a Merry Christmas and a Happy New Year!**