

8. Sheet for Numerics of Stationary Differential Equations

Exercise 23

- (a) Give the local linear basis functions for the reference triangle E_0 (reference element) with nodes $(0, 0)$, $(1, 0)$ and $(0, 1)$.
- (b) Give the corresponding local matrices

$$\int_{E_0} \phi_i \phi_j, \quad \int_{E_0} \partial_x \phi_i \partial_x \phi_j, \quad \int_{E_0} \partial_x \phi_i \partial_y \phi_j, \quad \int_{E_0} \partial_y \phi_i \partial_x \phi_j, \quad \int_{E_0} \partial_y \phi_i \partial_y \phi_j,$$

for $i, j = 1, 2, 3$.

Exercise 24

Give the affine transformation between an arbitrary triangle element $E \subset \mathbb{R}^2$ (with nodes (x_i, y_i) , $i = 1, 2, 3$) and the reference triangle E_0 . How does the inverse transformation look like?

Exercise 25

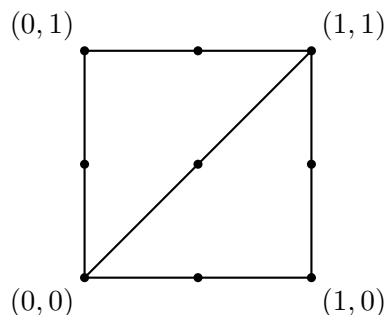
Transform, using the affine map from E to E_0 (see Exercise 24), the following integrals on E to integrals on E_0 :

$$\int_E \phi_i \phi_j, \quad \int_E \nabla \phi_i \cdot \nabla \phi_j.$$

Hint: Use the standard integral transformation (Jacobian) for affine maps.

Exercise 26

Give the basis functions for a triangle element with a quadratic polynomial space. Give the global basis function corresponding to the point $(\frac{1}{2}, \frac{1}{2})$ in the triangulation of the unit square shown below.



Explain how to obtain basis functions for a triangle element with cubic polynomials.

Solutions are discussed on Tuesday December 16, 2025

Tutor: Georgios Vretinaris - if you have question just come to my office (C3P16) or write me an email.