

5. Sheet for Numerics of Stationary Differential Equations (with Solutions)

Exercise 13

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain. How do you need to modify the finite differences method for the Poisson equation

$$-\Delta u = f \quad \text{in } \Omega, \quad u = g \quad \text{on } \Gamma$$

if Ω is not bordered by grid lines? Discuss this for a five-points star which goes through the boundary.

Is the matrix still symmetric? Does the discrete maximum principle still hold?

Exercise 14

Let u be the solution to the Poisson equation with Dirichlet boundary conditions:

$$-\Delta u = f \quad \text{in } \Omega, \quad u = g \quad \text{on } \Gamma \quad (*)$$

and let \tilde{u} solve the problem for perturbed boundary data \tilde{g} . Assume $u, \tilde{u} \in C^2(\Omega) \cap C(\bar{\Omega})$. Show:

$$\sup_{\Omega} |\tilde{u} - u| \leq \sup_{\Gamma} |\tilde{g} - g|$$

and further show a similar bound for the finite difference approximation.

Exercise 15

Let u be the solution to (*) and let \tilde{u} be the solution to the problem with perturbed right hand side \tilde{f} . Assume $u, \tilde{u} \in C^2(\Omega) \cap C(\bar{\Omega})$. Show:

$$\sup_{\Omega} |\tilde{u} - u| \leq \frac{r^2}{4} \cdot \sup_{\Omega} |\tilde{f} - f|,$$

if Ω is contained in a circle of radius r .

Hint: You already know this for the discretized problem.

Programming Exercise 3 :

Solve approximately

$$-\Delta u = 1 \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \Gamma$$

using the finite differences method on the unit square Ω - or if you want also on the unit circle. Choose the mesh size $h = 1/16$.

Solutions are discussed on Tuesday November 25, 2025

Tutor: Georgios Vretinaris - if you have question just come to my office (C3P16) or write me an email.