Universität Tübingen
Mathematisches Institut
Dr. Yanyan Shi
Dr. Yoann Le Hénaff

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7. Exercise Sheet for Algorithms in Numerical Mathematics

Exercise 20:

Give an algorithm which computes the QR-step, i.e. (just compute the R factor as the Q takes at least $O(n^2)$) of a symmetric, tridiagonal matrix of dimension n in $O(n)$.

Exercise 21: (Francis QR-step)

In the algorithm for the computation o f complex eigenvalues of real matrices, presented in the lecture, one uses the first column of the matrix M_k.

(a) Give an algorithm that computes $M_k e_1$ in as few operations as possible.

(b) Then give an algorithm that computes the reflection $Q(M_k e_1) = \alpha e_1$ with Householder matrix Q as efficiently as possible.

Exercise 22:

Prove that the QR-decomposition of a symmetric tridiagonal matrix A, is such that Q is in Hessenberg form and R is an upper triangular banded matrix with bandwidth 2, i.e.

$$R = \begin{pmatrix} * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{pmatrix}$$

Programming Exercise 5: Implement the algorithm of Exercise 20 and test it for at least one matrix.

Solutions are discussed on Tuesday 03.06.2025.
Tutor: Georgios Vretinaris - if you have question just come to my office (C3P16) or write me an email.